Background

This document provides a detailed description of the calculations underlying the Collection Volume Method (CVM) for positioning air terminals on structures and around sites and facilities for protection against cloud-to-ground lightning discharges.

Most of the theoretical and scientific background of the CVM is covered in considerable detail in the published journal literature (D’Alessandro et al 2001, 2003). Therefore, this document will focus mainly on the computational aspects.

Readers should understand from the outset that the more advanced scientific nature of the CVM means it cannot be implemented with trivial calculations involving simple analytical formulae. The collection volume of each point of interest is determined using numerical and iterative calculations, i.e., computations are carried out for different vertical and lateral positions of the downward leader.

Hence, the overall aim of this document is to provide a traceable procedure for numerical calculations of the collection volume and attractive radius of a specified point.

Review of the key points of the CVM

Eriksson’s (1979, 1980, 1987) improved EGM took a more physical approach than the simple EGM or RSM by taking into account the dependence of striking distance on the structure height in addition to the known dependence on peak stroke current (or downward leader charge) shown in Table 1 below per IEC 62305-1 (2006).

Table 1: Summary of the key parameters and probabilities associated with the lightning protection levels used to design an LPS for ordinary structures.

<table>
<thead>
<tr>
<th>LPL</th>
<th>Leader charge Q (C)</th>
<th>Peak current I₀ (kA)</th>
<th>Striking distance (m)</th>
<th>% strikes &gt; I₀ (Interception Efficiency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.16</td>
<td>2.9</td>
<td>20</td>
<td>99</td>
</tr>
<tr>
<td>II</td>
<td>0.38</td>
<td>5.4</td>
<td>30</td>
<td>97</td>
</tr>
<tr>
<td>III</td>
<td>0.93</td>
<td>10.1</td>
<td>45</td>
<td>91</td>
</tr>
<tr>
<td>IV</td>
<td>1.80</td>
<td>15.7</td>
<td>60</td>
<td>84</td>
</tr>
</tbody>
</table>
In considering height as an important variable, Eriksson’s model allowed for the electric field intensification created by the structure. The degree of field intensification is defined by the “field intensification factor”, $K_i$. Hence, the improved striking distance relationship presented by Eriksson was

$$d_s = f (I_p, K_i).$$

The extension of the Eriksson’s Improved EGM into a practical, three-dimensional air terminal placement method is referred to as the “Collection Volume Method” (CVM).

For structures, the $K_i$ is determined to a large extent by the height and width, but the shape and radius of curvature of the structure or structural features are also important. In the case of vertical air terminations, including freestanding masts, the $K_i$ depends on the height and tip radius of curvature, as has been demonstrated in numerous papers (e.g., D’Alessandro 2003). For horizontal air terminations, including shield wires, similar concepts are applied. In addition, when air terminations are positioned on structures, the $K_i$’s are multiplied up by a factor that depends on the structure dimensions and their location on the structure (D’Alessandro 2003).

Hence, an improved lightning protection design concept is to assume all points on a structure are able to launch an intercepting upward leader, but to differentiate those points based on the spatial electric field and degree of enhancement. This value, at any point in space, $K_i(x,y,z)$, is computed relatively easily using numerical techniques such as the finite element method. Fig. 1 shows an example of the output from such an analysis.

![Figure 1: Example of electric field analysis (in this case, using the FEM).](image)

The CVM considers the approach of the lightning downward leader to a structure and, using the $K_i(x,y,z)$ of the air terminations and the structure plus any other points of interest (the “competing features”), determines the point at which an upward leader will be launched from each location. Eriksson’s original model used the landmark work of (Carrara & Thione 1976) to determine leader inception. The critical radius concept of these authors is what was implemented in the CVM.

As an additional criterion, the CVM stipulates that interception will only occur if an adjacent competing feature does not “win the race” to interception with the downward leader. This criterion introduces a “time” variable which is taken into account by the ratio of downward and upward leader velocities, $K_v$. From field observations of natural lightning, the median value for this ratio is typically of the order of unity (Yokoyama et al 1990, Miyake 1994).

The above analysis typically produces a parabolic-like volume above the prospective strike point, as illustrated in Fig. 2. This volume represents the three dimensional “capture” or “collection volume” of that point. For a particular leader charge and velocity ratio, a downward leader will only terminate at the nominated point if the striking distance is attained (the red surface of Fig. 2a) and the leader path is contained within the velocity- or propagation-related boundary (the blue surface in Fig. 2a) of the collection volume. Collection volumes are calculated for all points of interest around the facility or site.

The collection volume information is summarised in the form of an “attractive radius”, $R_{attract}$, which is simply the sectional radius of the collection volume, i.e., radius at the intersection point of the striking distance surface and velocity-limited boundary. The attractive radius is an important output parameter of a collection volume analysis as it is used to compute the attractive or capture area of the point of interest on the structure, facility or site, regardless of whether it is an air termination or a competing feature. When the collection volumes of all points have been computed, the attractive areas are compared to determine whether the pre-specified protection level has been achieved.
Hence, it can be seen that two key features of the CVM result in significant advancements over previous EGM’s. That is, the CVM:

[1] Requires extensive electric field modeling (in 3D) to be carried out ….
  - Greater weight is given to taller air terminations;
  - Structure dimensions, particularly height and width, are taken into account; and
  - Physical criteria for leader inception must be met.

[2] Enforces the important concept of “competing features” …
  - **All** points are considered capable of launching upward leaders and hence must be taken into account in the analysis.

The next section describes four further aspects of the CVM that must be understood prior to implementation of the method, namely:

- How to determine the instant of the launch of an upward leader;
- Quantitative corrections for the atmospheric conditions affecting ionization processes in air;
- Leader velocity considerations; and
- Allowance for tall structures that may be subjected to so-called “side strikes”.

### Specific aspects

**a) Criterion for upward leader initiation**

The leader inception criterion employed by Eriksson (1979) was based on the landmark research of the time – the “critical radius” (CR) concept of Carrara & Thione (1976). Subsequently, the CR concept was used in all CVM implementations, so further information is needed here to complete the description. In the years after the CR concept was developed by Carrara & Thione, many other researchers have used it, e.g., Dellera & Garbagnati (1990) in their “leader progression model”.

The CR concept is a common way of dealing with corona and the rounding effect it has on sharp protrusions. In terms of this concept, the initiation of a stable leader requires the attainment of the corona threshold field (~ 3 MV/m at S.T.P.) over a “critical corona radius”. This means that corona inception at the tip of a ground-based point is not sufficient – the ambient field must continue to increase, thereby increasing the radius of the corona sheath around the point. Eventually, the threshold field is attained at a sufficiently large radius around the point and an upward leader is initiated. From a series of laboratory measurements using large air gaps, Carrara & Thione (1976) found this critical radius to be approximately 0.38 m for vertical rods. This means that any “sharp” geometrical features such as the tips of lightning rods, building corners, edges etc. with a radius less than the critical value must be “rounded off” to the relevant value.
Many years after the landmark paper of Carrara & Thione (1976), Bernardi et al (1996) carried out further experiments on the critical radius. They found somewhat smaller critical radii, approximately 0.28 m for vertical rod. Therefore, in the standard CVM implementation, in order to take a conservative approach, geometrically sharp features on structures such as corners and edges, i.e., all competing features, are modelled with the smaller radii of curvature values, while the tips of air terminals are geometrically rounded to critical radii of 0.38 m. Hence, the electric field field computations involve the estimation of the $K_i$ values at the surface of these artificially curved features. In terms of the CR concept, the attainment of ~ 3 MV/m at the surface of this “imaginary sphere” is taken to be the air breakdown condition for upward leader inception.

(b) Air breakdown corrections

The electric field strength required for air breakdown varies directly with air pressure (or density) and humidity. The dependence on humidity is relatively small and can generally be ignored. However, the dependence on air pressure is significant. For a decrease in air pressure, there is a decrease in the critical breakdown field. This affects the value used in the calculations dealing with leader inception upon the approach of the downward leader.

Hence, the electric field strength for air breakdown over the critical radius must be corrected for variations in air pressure. A simple linear approximation can be used to apply such a correction, e.g.,

$$E_b(z) = E_0 [0.82 + 0.18 (1 – 0.5z)] \quad (2)$$

(c) Leader velocity considerations

The attainment of the upward leader inception field is a necessary but not sufficient condition for lightning attachment – it does not guarantee interception. Eriksson (1979, 1987) states that the lightning flash will terminate on the structure rather than the ground provided the upward leader traverses the intervening distance before the critical breakdown field conditions are reached at the ground. Referring to Fig. 3, let $z_m$ be the distance between the downward leader tip and point B when the field at B reaches the value required for leader initiation. Eriksson (1979, 1987) took the most conservative approach by stipulating that the upward leader must traverse the full distance ($d_s – z_m$), to intercept the downward leader, before the latter has advanced the distance ($z – z_m$) toward the ground. Defining the average upward leader velocity $v_u$, this criterion reduces to

$$\frac{(d_s - z_m)}{v_u} \leq \frac{(z - z_m)}{v_d} \quad (3)$$

Hence, the leader velocity ratio, $K_v = \frac{v_d}{v_u}$, is also an important parameter in the CVM.

![Figure 3: Model of downward leader approach to a structure at some arbitrary lateral distance.](image-url)
Since \( d_s^2 = (z - H)^2 + d^2 \) from Fig. 3, substituting for \( d_s \) in Eqn. (3) gives:

\[
\frac{z}{d} = \sqrt{\frac{2z(z - H^2)}{K - 1} + \frac{2z}{K - 1} - (z - H)^2}.
\]

Hence, for any given point at a height \( H \), the points \((d, z)\) satisfying Eqn. (4) trace out a parabolic-like volume. These points must be obtained through a set of iterative calculations for different lateral displacements of the downward leader.

\(d\) Treatment of “tall” structures

To reduce the probability of strikes to sub-structures near ground level and within the assumed shielding zone of a tall structure (defined as \( \geq 60 \text{ m} \) by the IEC standard), a “de-rating angle” is applied to the collection volume attractive radius. An apparently safe overlap of capture areas from a plan view is “de-rated” due to the excessive vertical separation of capture areas. Table 2 shows the de-rating angles used in the past with the CVM for the four LPL’s. These angles were based on the results of the field studies by Gorin et al. (1976) of strikes to the Ostankino TV tower in Moscow.

**Table 2:** De-rating angles applied in the CVM for tall structures (\( H \geq 60 \text{ m} \)).

<table>
<thead>
<tr>
<th>LPL</th>
<th>Interception Efficiency</th>
<th>De-rating Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>99</td>
<td>26˚</td>
</tr>
<tr>
<td>II</td>
<td>97</td>
<td>23˚</td>
</tr>
<tr>
<td>III</td>
<td>91</td>
<td>20˚</td>
</tr>
<tr>
<td>IV</td>
<td>84</td>
<td>15˚</td>
</tr>
</tbody>
</table>

In the next section, the computation procedure for the CVM is described in a step-by-step manner.

**CVM computation procedure**

Using the crucial information summarised above, the overall CVM calculation procedure is as follows:

I. Specify all elevated objects heights, widths and shape, and any structural features.

II. Identify the “most probable” competing features (outer, sharper features).

III. Select the number, location and height of the air terminations (using a rough estimate of the attractive area of each).

IV. Specify the basic physical parameters:

- Downward leader charge / prospective peak current / protection level, as per Table 1 or similar;
- Cloud base height;
- Site elevation or altitude above sea level and apply the appropriate correction factor to the air breakdown field if applicable;
- Leader velocity ratio; and
- Field intensification factors for all air terminations and competing features.

V. For all air terminations and nominated competing features, compute the:

- Collection volume (striking distance surface, using the critical radius concept, and the velocity / leader propagation-based boundary);
- Attractive radius from the intersection point of the striking distance surface (for the given leader charge / LPL) and the velocity boundary.

VI. If the structure has a height greater than or equal to 60 m, apply the appropriate collection volume de-rating angle (Table 2).

VII. Apply the attractive radii or areas to their respective air terminations and competing features.

VIII. Check to see if the air termination capture areas completely overlap the attractive areas of all competing features (a plan view is useful here).

XI. If there is not complete overlap, use more air terminations, or relocate some of the existing ones, and repeat the above steps until complete overlap is achieved.
There are various components to the CVM calculations, so these are described below in a modular format.

(a) Downward leader / ambient field – spatial & temporal development

The modelling of the downward leader approach in the CVM is implemented using Eriksson’s (1979) analytical relation based on an assumed linearly distributed charge on the downward leader, viz.

\[
E_A = \frac{Q}{\pi \varepsilon d^2} \left[ \frac{(h - z)^2}{(h - z)^2 - (z - d)^2} + \sinh^{-1} \left( \frac{z - d}{d} \right) - \sinh^{-1} \left( \frac{h - z}{d} \right) \right]
\]

where \( E_A \) is the electric field at point A (top of the structure) in Fig. 3 in the absence of the structure (the field enhancement due to the presence of the structure is dealt with in the next step). The value of \( Q \) for the total charge on the downward leader (which is distributed linearly along its length) is obtained from Table 1 for the pre-specified LPL. The length of the downward leader is defined by the cloud base height, \( h \), which is a user-defined parameter. It is recommended that a cloud base height of \( h = 5000 \) m is used as the default value.

Eqn. (5) reduces to a more simple form when the lateral displacement \( d = 0 \). In Fig. 3, the field at point B on the ground directly below the downward leader is given by:

\[
E_B = \frac{Q}{\pi \varepsilon (h - z)^2} \left[ \frac{h - z}{z} + \ln \left( \frac{z}{h} \right) \right]
\]

The purpose of this step in the CVM analysis is to obtain the striking distance surface. For each lateral (horizontal) position of the downward leader, starting at the central position or “axis of symmetry”, a series of vertical iterative steps are needed to simulate the downward approach, as shown in Fig. 4.

(b) Geometric e-field around ground objects – spatial distribution

Numerical computations are required to determine the degree of electric field intensification, \( K_i \), created by structures and other grounded objects and points in the presence of the ambient field due to the thundercloud and approaching downward leader. The product of \( K_i \) at a point in space and the ambient field at that point is a good approximation of the overall magnitude of the e-field existing at that point in space and it is this value that is used to determine the instant of upward leader initiation.

The computations of the spatial field distribution must be carried out with all geometrical features and objects bloated to the critical radius. For conservatism, competing features and air terminations should be allocated different critical radii – 0.28 m for most competing features and 0.38 m for vertical masts / rods. Once the electric field intensification over the critical radius has been determined, it is a trivial matter to determine the instant of upward leader initiation as part of the vertical iterations mentioned in (a) above. Figure 4: Illustration of the iterative calculations (vertical and horizontal) needed to define the striking distance surface of the collection volume.

(c) Velocity-derived boundary & attractive radius determination

The velocity-derived boundary is also obtained via an iterative calculation, using Eqn. (4). Once the velocity boundary has been defined, the attractive radius can be obtained from the intersection of the velocity boundary and the striking distance surface.
Figure 4: Illustration of the iterative calculations (vertical and horizontal) needed to define the striking distance surface of the collection volume.

(d) De-rating Angles

It must be remembered that the de-rating angles given in Table 2 are very rough guidelines and are simply a means to cater for the possibility of strikes near the base of a tall structure, e.g., where there may be other low-lying structures that are vulnerable and/or people walking around that may be exposed to a strike due to the branching of a downward leader or an oblique trajectory. Figure 5 illustrates how the de-rating angles are applied to the collection volume analysis.

(e) Determination of Protection Status

After the collection volumes have been computed for all protective air terminals and all of the competing features around the building, facility or site, the final step in the calculation involves the determination of the protection status.

Figure 6 illustrates the usefulness of a “2D plan view” display of the results for this purpose. The building or site is protected if and only if the air termination attractive areas (shown in light blue) completely overlap all of the competing feature attractive areas (darker blue when shielded by the air terminations and red if exposed to lightning).

Figure 5: Application of collection volume de-rating to tall structures (example for LPL III). Note that, in this particular example, the lower structure would not have been deemed to be protected anyhow, since the collection volume of the air termination on top of the main building does not overlap the small collection volume of the corners of the lower structure.

Figure 6: 2D plan view display of the three air termination attractive areas (light blue) and the competing features attractive areas (darker blue), including those not shielded or protected by the air terminations (red areas).
References


